Exercise 23

Find the gradient vector field of f.

$$f(x,y,z) = \sqrt{x^2+y^2+z^2}$$

Solution

Calculate the gradient and call it $\mathbf{F}$.

$$\begin{aligned}
\mathbf{F} &= \nabla f \\
&= \left\langle \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right\rangle f \\
&= \left\langle \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z} \right\rangle \\
&= \left\langle \frac{\partial}{\partial x}\left(\sqrt{x^2+y^2+z^2}\right), \frac{\partial}{\partial y}\left(\sqrt{x^2+y^2+z^2}\right), \frac{\partial}{\partial z}\left(\sqrt{x^2+y^2+z^2}\right) \right\rangle \\
&= \left\langle \frac{1}{2}(x^2+y^2+z^2)^{-1/2} \cdot \frac{\partial}{\partial x}(x^2+y^2+z^2), \frac{1}{2}(x^2+y^2+z^2)^{-1/2} \cdot \frac{\partial}{\partial y}(x^2+y^2+z^2) \right. \\
&\qquad \left. , \frac{1}{2}(x^2+y^2+z^2)^{-1/2} \cdot \frac{\partial}{\partial z}(x^2+y^2+z^2) \right\rangle \\
&= \left\langle \frac{1}{2}(x^2+y^2+z^2)^{-1/2} \cdot (2x), \frac{1}{2}(x^2+y^2+z^2)^{-1/2} \cdot (2y), \frac{1}{2}(x^2+y^2+z^2)^{-1/2} \cdot (2z) \right\rangle \\
&= \left\langle \frac{x}{\sqrt{x^2+y^2+z^2}}, \frac{y}{\sqrt{x^2+y^2+z^2}}, \frac{z}{\sqrt{x^2+y^2+z^2}} \right\rangle
\end{aligned}$$